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Computers & Chemical Engineering

Computers and Chemical Engineering 28 (2004) 811-819

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# A comparison of the feedback control behavior between thermally coupled and conventional distillation schemes

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#### **Abstract**

A comparative analysis of the feedback control responses to set point changes of three thermally coupled distillation schemes and two conventional distillation sequences for the separation of ternary mixtures is presented. Designs for the thermally coupled schemes were obtained and optimized for energy consumption to link their energy characteristics to their dynamic behavior. For the comparison of the dynamic behavior, responses to set point changes under closed loop operation with proportional-integral (PI) controllers were obtained. For each separation scheme, the parameters of the PI controllers of the three composition control loops were optimized using the integral of the absolute error criterion. The effects of feed composition and of the ease of separability index were considered. The dynamic tests showed that in many cases the thermally coupled distillation schemes outperformed the dynamic responses of the conventional distillation sequences. The results indicate that there exist cases in which the energy savings provided by the thermally coupled systems do not conflict with their dynamic properties. © 2004 Elsevier Ltd. All rights reserved.

Keywords: Distillation; Thermally coupled distillation; Distillation control; Energy integration

#### 1. Introduction

Distillation processes, which constitute the most commonly used method to separate liquid mixtures in the chemical industry, are characterized by its high energy consumption. Alternate arrangements to sequences based on conventional distillation columns (one input, two outputs) have received noticeable attention in recent years. Through the use of recycle streams between two columns, several thermally coupled distillation systems have been proposed. For the case of the separation of ternary mixtures, three thermally coupled distillation systems (TCDS) have been particularly analyzed: the system with side rectifier (TCDS-SR, Fig. 1a), the system with side stripper (TCDS-SS, Fig. 1b), and the fully thermally coupled distillation system (or Petlyuk column, Fig. 1c). Several studies have indicated that those TCDS schemes can save up to 30% on energy consumption with respect to the conventional

Fig. 1 (Agrawal & Fidkowski, 1998, 1999). In particular,

it has recently been shown that several alternative systems

direct (one-by-one as overheads) and indirect (one-by-one as bottoms products) sequences shown in Fig. 2 (Alatiqi &

Luyben, 1985; Fidkowski & Krolikowski, 1990; Glinos &

Malone, 1988; Tedder & Rudd, 1978; Yeomans &

Grossmann, 2000). Most of these results were obtained

through energy consumption calculations at minimum reflux conditions, and they spawned the development of more formal design procedures. Hernández and Jiménez (1996, 1999a) have reported the use of optimization strategies for TCDS to detect designs with minimum energy consumption. When comparing the energy savings of the integrated schemes, it has been found that in general the Petlyuk system offers better savings than the systems with side columns. However, the complex column configurations that can potentially produce larger energy savings are not commonly used in industrial practice, largely because of control concerns (Dunnebier & Pantelides, 1999). The uncertainty of the control properties of TCDS has promoted the search for new schemes with a reduction in the number of thermally coupled interconnections, which in principle should improve the dynamic properties of the original systems of

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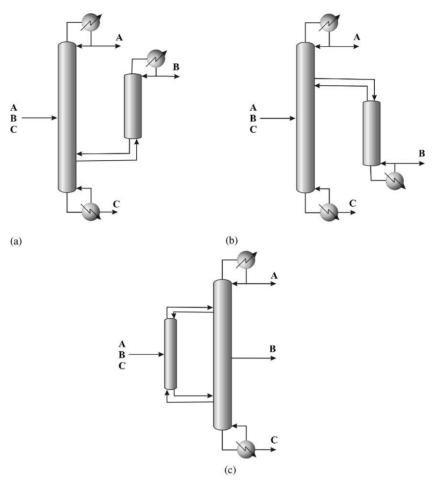


Fig. 1. Different thermally coupled distillation arrangements: (a) TCDS-SR, (b) TCDS-SS, (c) Petlyuk column.

to the Petlyuk system provide similar energy savings as the original scheme (Jiménez, Ramírez, Castro, & Hernández, 2003). On the same lines, Rong and Kraslawski (2002, 2003) have reported the generation of thermally coupled distillation schemes with lower number of interconnecting streams.

Some research efforts have also been conducted to understand the operational properties of TCDS. The works of Wolff and Skogestad (1995) Abdul Mutalib and Smith (1998), Hernández and Jiménez (1999b) and Jiménez, Hernández, Montoy, and Zavala-García (2001) have shown that some of these integrated options are controllable, so that their potential implementation would probably not be at the expense of control problems. Serra, Espuña, and Puigjaner (2003) have shown that the theoretical control properties of the Petlyuk column can be improved significantly through the operation at energy consumption levels slightly different from the optimum design.

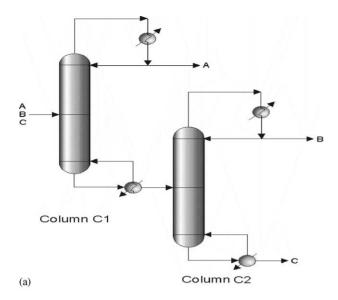
Following the promising results on energy savings of coupled schemes for ternary mixtures, some researchers have started to study the separation of mixtures of more than three components. Christiansen et al. (1997) extended the ideas developed for the Petlyuk system to the separation of quaternary mixtures. Rong, Kraslawski, and Nyström (2000) pre-

sented an analysis on thermally coupled systems with side columns for the separation of mixtures with five components, and showed that these arrangements provide potential economic savings over conventional sequences.

In this work, we present an analysis on the energy requirements and the closed loop behavior of the three TCDS of Fig. 1, and compare their energy consumption and dynamic responses to those of the conventional direct and indirect distillation sequences.

# 2. Design procedure

The design of the three TCDS under consideration was conducted following the method reported by Hernández and Jiménez (1996, 1999a). The method provides first a tray structure for the integrated systems by a section analogy procedure with respect to the design of a conventional sequence; the TCDS-SR is obtained from the tray arrangements of a direct sequence, the TCDS-SS from an indirect sequence, and the Petlyuk system from a sequence of a prefractionator followed by two binary distillation columns. Then, the degrees of freedom that remain after design specifications (one degree of freedom for the systems with side columns,



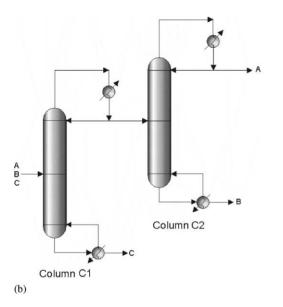


Fig. 2. Conventional distillation sequences. (a) Direct sequence, (b) indirect sequence.

and two for the Petlyuk system) are used to obtain the operating conditions under which the integrated designs provide minimum energy consumption.

The search procedure provides the optimal values of the interconnecting vapor flowrate (VF) for the TCDS-SR (Fig. 1a), the interconnecting liquid flowrate (LF) for the TCDS-SS (Fig. 1b), or both interconnecting streams for the case of the Petlyuk column (Fig. 1c). The final design is tested through rigorous simulations (we used in this work Aspen Plus for that purpose.) The design is successful if it meets the product specifications; otherwise proper adjustments are made in the tray structure or reflux flowrate values until the design specifications are met. The dynamic analysis is then conducted using the validated design with minimum energy consumption for each case.

#### 3. Dynamic simulations and case studies

One of the key parts for the dynamic analysis is the selection of control outputs and manipulated variables for each control loop. Although more formal techniques to define the control loops for the integrated columns may be used (for instance the relative gain array method or the singular value decomposition technique), we based our selection on practical considerations. Thus, for any sequence, the control of the lightest component of the ternary mixture was manipulated with the top reflux flowrate, and the heaviest component with the reboiler heat duty. The control of the intermediate component, on the other hand, depended on the integrated structure under consideration; for the TCDS-SR it was paired to the reflux flowrate of the side rectifier, for the TCDS-SS to the heat duty of the side stripper, and for the Petlyuk column to the product stream flowrate. The closed loop analysis was based on proportional-integral controllers. Several alternatives exist for tuning up the controller parameters. We attempted a common ground for comparison by optimizing the controller parameters, proportional gains  $(K_{\rm C})$  and reset times  $(\tau_{\rm i})$ , for each conventional and integrated scheme following the integral of the absolute error (IAE) criterion. For the integrated arrangements, the procedure is particularly complicated because of the interactions of the multivariable control problem. For these cases, the tuning procedure was conducted taking one control loop at a time; the parameters thus obtained were taken for the following control loop until the three loops were considered.

The case studies were selected to reflect different separation difficulties and different contents of the intermediate component of the ternary mixtures. Three mixtures with different values of the ease of separability index (ESI, the ratio of relative volatilities of the split AB to the split BC, as defined by Tedder & Rudd, 1978) were considered. The selected mixtures were n-pentane, n-hexane and n-heptane (M1, ESI = 1.04), *n*-butane, isopentane and *n*-pentane (M2,ESI = 1.86), and isobutane, *n*-butane and *n*-hexane (M3, ESI = 0.18). To examine the effect of the content of the intermediate component, two types of feed compositions were assumed. One feed with a low content of the intermediate component (where mole fractions of A, B, C were equal to 0.40, 0.20, 0.40 feed F1) and another one with a high content of the intermediate component (A, B, C equal to 0.15, 0.70, 0.15 feed F2), were used. The total feed flowrate for all cases was 45.5 kmol/h. Product purities of 98.7, 98 and 98.6% for A, B and C, respectively, were assumed as part of the design specifications.

# 4. Energy requirements

The first part of the analysis was conducted to detect the designs with minimum energy consumption for the integrated sequences. The results presented were obtained after an optimization procedure was carried out on the

Table 1						
Energy requirements	(Btu/h)	for the	eseparation	of the	ternary	mixtures

Feed	Direct sequence	Indirect sequence	TCDS-SR	TCDS-SS	Petlyuk column
Mixture M1					
F1	3,263,772.2	3,547,190.0	2,521,007.0	2,730,465.2	1,709,474.1
F2	4,127,083.9	4,356,343.8	3,167,085	3,511,610.3	2,142,722.5
Mixture M2					
F1	7,430,812.6	7,277,925.1	7,106,695.5	6,895,831.3	6,300,486.6
F2	7,816,270.4	7,344,143.3	7,073,923.0	6,958,312.5	6,142,722.5
Mixture M3					
F1	3,887,458.3	5,697,594.6	2,952,695.0	3,045,980.6	2,399,649.4
F2	5,756,130.7	5,300,344.3	3,859,170.5	3,816,550.1	2,600,934.7

recycle streams for each of the three thermally coupled sequences.

Table 1 shows the energy requirements for each integrated scheme and conventional sequence for the three ternary mixtures and the two assumed feed compositions. When mixture M1 was considered, the Petlyuk system showed the best potential, offering savings in energy consumption of up to 50% with respect to the conventional distillation sequences. The TCDS-SR and TCDS-SS require between 14 and 20% less energy consumption than the conventional sequences.

The superior behavior on energy efficiency of the Petlyuk column was also observed for mixtures M2 and M3 (Table 1). In the case of mixture M2, the Petlyuk column can offer savings in energy consumption of up to 15% with respect to the conventional sequences, while the savings achieved by the TCDS-SR and TCDS-SS schemes are in the order of 10%. In the case of mixture M3, the Petlyuk column requires between 40 and 50% less energy consumption, whereas the TCDS-SR and the TCDS-SS options offer energy savings of up to 30% with respect to the conventional sequences. In general, the Petlyuk column offered the highest energy savings with respect to the direct and indirect conventional distillation sequences for all the case studies considered.

## 5. Dynamic results

For the dynamic analysis, individual set point changes for product composition were implemented for each of the three product streams. For all cases (conventional and integrated sequences), the three control loops were assumed to operate under closed loop fashion. The performance of the sequences under analysis was compared through the evaluation of IAE values for each test. This part of the study was conducted with the use of Aspen Dynamics.

#### 5.1. Mixture M1

Table 2 shows the IAE values obtained for each composition control loop of the five distillation sequences for mixture M1. When feed F1 was considered, the Petlyuk column offered the best dynamic behavior, based on the lowest val-

ues of IAE, for the control of the three product streams. The individual dynamic responses of each control loop for the five distillation sequences are displayed in Figs. 3–5. The control of the lightest component (Fig. 3) or the heaviest component (Fig. 5) does not create any significant problems for any of the five distillation sequences, although the Pelyuk column showed the lowest IAE values. However, one may notice how the conventional direct sequence is unable to control the composition of the intermediate component (Fig. 4), while the Petlyuk column provides a smooth response, with a relatively short settling time. It is interesting to notice that for this mixture with an ESI = 1 and a low content of the intermediate component in the feed, the Petlyuk column offers the highest energy savings and also shows the best dynamic performance from the five distillation sequences under consideration.

When the content of the intermediate component in the feed was raised from 20 to 70%, significant changes in the dynamic responses of the distillation systems were observed (feed F2, Table 2). The first remark is that the Petlyuk column is not the dominant choice from an operational point of view, although such a scheme provided the lowest IAE values for the control of the heaviest component. A second observation is that the best choice depends on the control loop of primary interest. When the control of the lightest component of the ternary mixture is of primary concern, then

Table 2 IAE results for mixture M1

Sequence	Component			
	A	В	С	
Feed F1				
Direct	$7.92441 \times 10^{-3}$	$5.28568 \times 10^{-2}$	$2.95796 \times 10^{-3}$	
Indirect	$4.0076 \times 10^{-3}$	$3.4576 \times 10^{-3}$	$2.64873 \times 10^{-3}$	
TCDS-SR	$3.55963 \times 10^{-3}$	$2.78147 \times 10^{-3}$	$7.99529 \times 10^{-4}$	
TCDS-SS	$7.69839 \times 10^{-4}$	$8.9876 \times 10^{-3}$	$3.80888 \times 10^{-4}$	
Petlyuk	$1.74924 \times 10^{-4}$	$3.42972 \times 10^{-4}$	$2.10607 \times 10^{-4}$	
Feed F2				
Direct	$8.06247 \times 10^{-4}$	0.00419091	$7.84564 \times 10^{-4}$	
Indirect	$7.84564 \times 10^{-4}$	$6.87839 \times 10^{-5}$	$4.90328 \times 10^{-4}$	
TCDS-SR	$3.06376 \times 10^{-5}$	0.00181494	$4.61142 \times 10^{-4}$	
TCDS-SS	$1.23953 \times 10^{-5}$	$1.23953 \times 10^{-4}$	$7.40356 \times 10^{-5}$	
Petlyuk	$4.53799 \times 10^{-5}$	$1.39297 \times 10^{-4}$	$1.89237 \times 10^{-5}$	

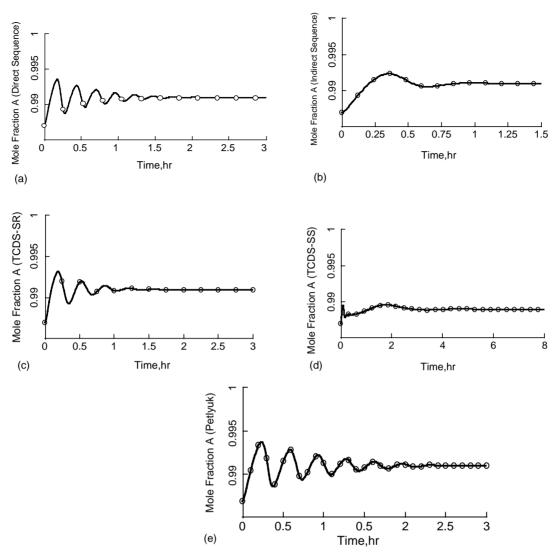


Fig. 3. Closed loop dynamic response for component A, mixture M1, feed F1. (a) Direct sequence, (b) indirect sequence, (c) TCDS-SR, (d) TCDS-SS, (e) Petlyuk column.

the TCDS-SS scheme provided the best option based on the lowest IAE value. On the other hand, if the control policy calls for the composition of the intermediate (B) component, the indirect sequence shows the best behavior, with the lowest value of IAE. Overall, it may be stated that for this type of mixture, the TCDS-SS and the Petlyuk column may offer good compromises, providing energy savings with respect to conventional sequences and good dynamic properties.

## 5.2. Mixtures M2 and M3

The analysis was completed with the consideration of the other four case studies. As a representative test, Fig. 6 shows the closed loop results for the five distillation sequences for mixture M3 with a low content of the intermediate component; all sequences provide proper dynamic responses, with the Petlyuk column showing the lowest IAE value. Tables 3 and 4 give the summary of IAE values for mixtures M2 and

Table 3 IAE results for mixture M2

Sequence	Component	iponent			
	A	В	С		
Feed F1					
Direct	0.00187781	0.010187	0.00144499		
Indirect	$1.6439 \times 10^{-4}$	$3.61135 \times 10^{-5}$	0.0094768		
TCDS-SR	$4.64586 \times 10^{-5}$	0.0053059	$3.2677 \times 10^{-4}$		
TCDS-SS	0.0128428	0.0244571	$8.92827 \times 10^{-4}$		
Petlyuk	$3.25178 \times 10^{-5}$	0.00164826	$1.2647 \times 10^{-4}$		
Feed F2					
Direct	$9.20186 \times 10^{-5}$	$6.65642 \times 10^{-5}$	$7.68231 \times 10^{-4}$		
Indirect	$1.9 \times 10^{-4}$	$3.44526 \times 10^{-4}$	0.01144		
TCDS-SR	$2.61485 \times 10^{-5}$	0.00528108	$1.70596 \times 10^{-5}$		
TCDS-SS	$9.95744 \times 10^{-5}$	0.0102255	0.00362916		
Petlyuk	0.00184601	0.00943306	0.0032212		

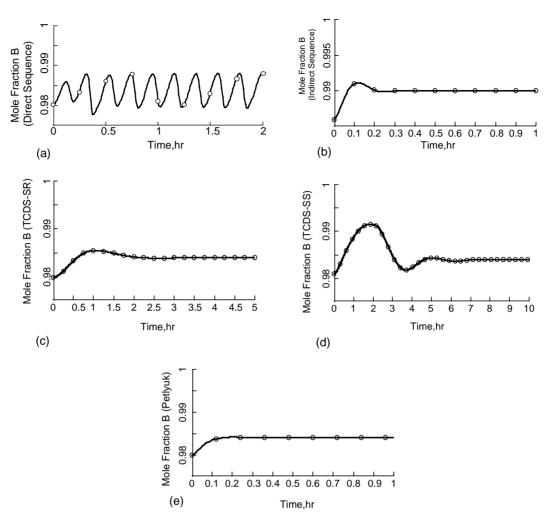


Fig. 4. Closed loop dynamic response for component B, mixture M1, feed F1. (a) Direct sequence, (b) indirect sequence, (c) TCDS-SR, (d) TCDS-SS, (e) Petlyuk column.

M3. Some trends were observed. For one thing, the best option depends on the amount of intermediate component. Also, it was found that the best sequence, based on the IAE criterion, for the control of the light component was

Table 4 IAE results for mixture M3

Sequence	Component			
	A	В	С	
Feed F1				
Direct	$1.73384 \times 10^{-4}$	$6.82658 \times 10^{-4}$	$5.77236 \times 10^{-5}$	
Indirect	$1.07421 \times 10^{-4}$	$3.96869 \times 10^{-5}$	$1.6405 \times 10^{-5}$	
TCDS-SR	$6.8821 \times 10^{-5}$	$2.88009 \times 10^{-4}$	$3.5794 \times 10^{-5}$	
TCDS-SS	$7.55231 \times 10^{-5}$	$1.36692 \times 10^{-4}$	$2.26436 \times 10^{-5}$	
Petlyuk	$8.439 \times 10^{-6}$	0.004537	$7.21853 \times 10^{-6}$	
Feed F2				
Direct	$5.39673 \times 10^{-4}$	$1.41202 \times 10^{-4}$	$3.02784 \times 10^{-5}$	
Indirect	0.00116516	0.00297374	$1.19592 \times 10^{-4}$	
TCDS-SR	$1.19562 \times 10^{-4}$	0.00340822	$5.23569 \times 10^{-6}$	
TCDS-SS	$2.52019 \times 10^{-4}$	$7.65563 \times 10^{-4}$	$9.91589 \times 10^{-5}$	
Petlyuk	$1.7815 \times 10^{-4}$	0.00269758	$2.21387 \times 10^{-5}$	

also the best choice for the control of the heavy component, but a different separation scheme provided the best option for the control of the intermediate component. In addition, the best dynamic performance is influenced by the feed composition. If the feed contains low amounts of the intermediate component (feed F1), the Petlyuk column shows the best dynamic behavior for the control of the light and heavy components, while the indirect sequence provides the best responses for the control of the intermediate component. For feed mixtures with a high content of the intermediate component (feed F2), the sequence with a side rectifier showed the best responses for the control of light and heavy components, while the conventional direct sequence provided the best choice for the control of the intermediate component. Interestingly, the only cases in which conventional sequences showed the best dynamic performance was for the control of the intermediate component; thermally coupled options, which have been expected to show adverse control properties, showed better dynamic responses for the control of the other components of the ternary mixture. Finally, for this set of mixtures

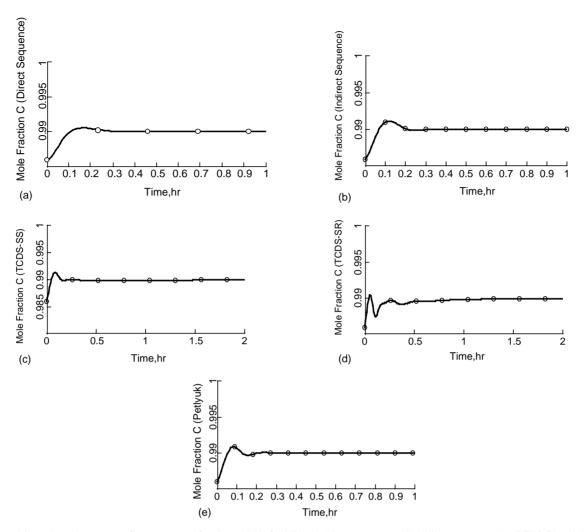


Fig. 5. Closed loop dynamic response for component C, mixture M1, feed F1. (a) Direct sequence, (b) indirect sequence, (c) TCDS-SR, (d) TCDS-SS, (e) Petlyuk column.

(M2 and M3, with ESI values different than one) and for feeds with high contents of the intermediate component, the sequences with two top distillate streams (TCDS-SR or the direct sequence) provided the best dynamic responses.

#### 5.3. Overall results

Table 5 summarizes the optimal options detected from the dynamic analysis for all case studies. Based on the trends

observed, a distinction is given between the best control option for extreme components of the ternary mixture (A and C) and the best scheme for the intermediate component (B). The influence of mixture and feed composition is also shown. The only case in which there was a dominant structure for all control loops was when the feed contained low amounts of the intermediate component and the mixture had an ESI value of 1, and, possibly unexpectedly, the Petlyuk column provided the optimal choice in such a case.

Table 5
Sequences with best dynamic responses for each control loop

Mixture	Feed with low content of intermediate component		Feed with high content of intermediate component		
	Control of A and C	Control of B	Control of A and C	Control of B	
M1	Petlyuk	Petlyuk	TCDS-SS, Petlyuk	Indirect	
M2	Petlyuk	Indirect	TCDS-SR	Direct	
M3	Petlyuk	Indirect	TCDS-SR	Direct	

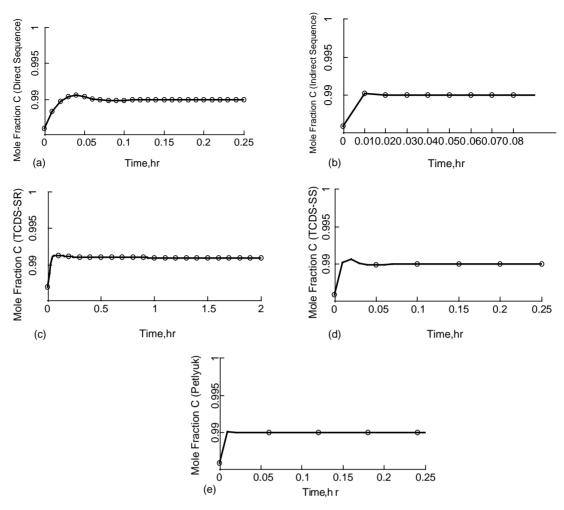


Fig. 6. Closed loop dynamic response for component C, mixture M3, feed F1. (a) Direct sequence, (b) indirect sequence, (c) TCDS-SR, (d) TCDS-SS, (e) Petlyuk column.

# 6. Conclusions

We have conducted a comparison on the dynamic behavior of five distillation sequences for the separation of ternary mixtures. An analysis on energy consumption complemented the analysis. Three of the sequences considered make use of thermal coupling, and their energy and control properties have been compared to those of the conventional direct and indirect sequences. From energy considerations the Petlyuk column shows generally the highest savings. The dynamic analysis was based on PI controllers, for which the parameters were tuned up through a minimization procedure of the integral of the absolute error. The results from the dynamic analysis do not show a dominant option, but interesting trends were observed. Two factors seem to affect the optimal choice from dynamic considerations. One is the amount of intermediate component, and the other one is the preferred control policy, i.e. which component of the ternary mixture is the most important from operational or marketing purposes. When the control of the lightest or heaviest component is of primary interest, integrated sequences provide interestingly the best options. When the amount of intermediate component is low, the Petlyuk column provided the best dynamic performance; when the amount of intermediate component is high, the integrated sequences with side columns showed the best dynamic results. On the other hand, when the control of the intermediate component is the desired strategy, the energy savings provided by the integrated sequences conflict with their control properties, since the conventional sequences offered generally the best dynamic responses. In summary, although the best operational option is not unique, the results show that there are cases in which integrated sequences do not only provide significant energy savings with respect to the conventional sequences, but also may offer some dynamic advantages.

## Acknowledgements

Financial support was provided by Conacyt and from Concyteg, Mexico, for the development of this project.

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