Planning of intensified production of solar grade silicon to yield solar panels involving behavior of population

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Abstract

The need to use clean sources for producing electricity has been a great topic of discussion in recent years. Many countries have begun to take advantage of solar energy through the production of solar panels for electric power generation. In addition, knowing people’s preferences allows predicting their behavior to propose better planning at macroscopic level. This work presents a mathematical programming model to address the conduct of people. The objective is to know their inclinations using the matching law, which involves the actions of users through different scenarios considering economic incentives and punishments. It includes a strategic planning of the production and distribution of solar grade silicon that is used for the construction of solar panels to meet the demands of electricity in residential sector of Mexico as a case study. Process intensification was used to enhance different ways to obtain solar grade silicon such as an Intensified Fluidized Bed Reactor and a hybrid, which is a combination of Siemens and conventional FBR processes. Also, the Siemens process was considered the most common process to produce silicon. Results show that the difference between the analyzed scenarios lies in the behavior of people while profit maintains constant.

Keywords: Matching law, Solar grade silicon, Solar panels, Strategic planning, Intensified silicon production process, Economic incentives

1. Introduction

The collection of solar energy through photovoltaic (PV) technology is increasingly recognized as an essential component of future global electricity generation. The depletion of fossil fuels, in addition to the detrimental effects of their excessive use, such as CO₂ emissions and some other greenhouse gases in the atmosphere, are driving research and the evolution of new energy sources that try to be more environmentally friendly [1].

In recent decades, photovoltaic systems have evolved and have been adapted to various applications of daily life. That is why the large-scale manufacturing of photovoltaic cells, capable of providing energy, is increasingly economically viable. Cost reductions are required in the production of raw materials, such as the case of one of the most used materials in the photovoltaic industry, the polysilicon. However, it is important to produce large amounts of polysilicon due to the high demand by the solar sector. The production of polysilicon from quartz consists of two main stages: obtaining metallurgical grade silicon and purifying it to transform it into solar grade silicon [2]. Attention has been drawn to update processes to obtain this component in order to achieve not only cost reduction but also improving safety of the process. Process intensification enhances safety through reduction of inventory, develop reactor/yield, minimizing feedstock, etc. [3]. A reliable definition of process intensification is “Any chemical engineering development that leads to a substantially smaller, cleaner and more energy efficient technology” [4]. There has been an increasing growth in the field of process intensification in the last few decades that featured successful industrial applications and increased interest in research [5]. In the process industry, sustainability and competitiveness are essential objectives and the implementation of process intensification helps to achieve a more sustainable and economically stronger process [6]. Process intensification leads to substantially cheaper processes, investment costs, costs of raw materials, costs of utilities, and costs of waste-stream processing [7], and it enhances physical processes including heat, mass, and momentum transfer [8].

Due to the exceeding cost of photovoltaic panels, it is necessary to look for alternatives in order to reduce the production cost of the solar grade silicon (Si₉₃). The photovoltaic industry relies on high-purity silicon produced in the Siemens process; however, it requires a high power...
Nomenclature

Sets

F  Production plants by FBR Union Carbide process
H  Production plants by Hybrid process
PC  Production plants of C
PH  Production plants of H₂
PHC  Production plants of HCl
PIN  Photovoltaic solar systems interconnected with the Network
PIS  Photovoltaic solar systems isolated
PS  Production plants of SiCl₄
PSI  Production plants of SiCl₄
S  Production plants by Siemens process
T  Time

Variables

Bₜ₂₈  Answer of population when isolated solar panel is used
Bₜ₂₈  Answer of population when interconnected solar panels are used
Costₘ-C  Production cost of C
Costₘ-FBR  Production cost of silicon in FBR process
Costₘ-H₂  Production cost of H₂
Costₘ-HCl  Production cost of HCl
Costₘ-Hyb  Production cost of silicon in Hybrid process
Costₘ-SiCl₄  Production cost of SiCl₄
Costₘ-SiO₂  Production cost of SiO₂
Costₙ-rw  Transport cost of raw material
Costₙ-SI  Transport cost of silicon
EPₜ₂₈  Electricity generated from interconnected solar panels
Eₜ₂₈  Electricity generated from isolated solar panels
Fₜ₂₈-SI  Installation capacity required to produce silicon in Siemens process
Fₜ₂₈-SiO₂  Installation capacity required to produce SiO₂
Fₜ₂₈-C  Flowrate of C produced
Fₜ₂₈-C-FBR  Flowrate of C needed in FBR process
Fₜ₂₈-C-Hyb  Flowrate of C needed in Hybrid process
Fₜ₂₈-C-SiCl₄  Flowrate of C needed in Siemens process
Fₜ₂₈-FBR  Flowrate of silicon produced in FBR process
Fₜ₂₈-FBR-SI  Flowrate of silicon produced in FBR process needed for interconnected solar panels
Fₜ₂₈-FBR-SI-SI  Flowrate of silicon produced in FBR process needed for isolated solar panels
Fₜ₂₈-H₂  Flowrate of H₂ produced
Fₜ₂₈-H₂-FBR  Flowrate of H₂ needed in FBR process
Fₜ₂₈-H₂-Hyb  Flowrate of H₂ needed in Hybrid process
Fₜ₂₈-H₂-SiCl₄  Flowrate of H₂ needed in Siemens process
Fₜ₂₈-HCl  Flowrate of HCl produced
Fₜ₂₈-HCl-SI  Flowrate of HCl needed in Siemens process
Fₜ₂₈-Hyb  Flowrate in Hybrid process
Fₜ₂₈-Hyb-SI  Flowrate of silicon produced in Hybrid process
Fₜ₂₈-SiCl₄  Flowrate of SiCl₄ produced
Fₜ₂₈-SiCl₄-FBR  Flowrate of SiCl₄ needed in FBR process
Fₜ₂₈-SiCl₄-Hyb  Flowrate of SiCl₄ needed in Hybrid process
Fₜ₂₈-SiCl₄-SI  Flowrate of silicon needed in isolated solar panels
Fₜ₂₈-SiCl₄-SI-SI  Flowrate of silicon needed in Siemens process
Fₜ₂₈-SiO₂  Flowrate of SiO₂ produced
Fₜ₂₈-SiO₂-FBR  Flowrate of SiO₂ needed in FBR process
Fₜ₂₈-SiO₂-Hyb  Flowrate of SiO₂ produced in Siemens process
Flowrate of silicon produced in Siemens process
Flowrate of silicon produced in Siemens process needed for interconnected solar panels
Flowrate of silicon produced in Siemens process needed for isolated solar panels
Flowrate of SiO₂ produced
Flowrate of SiO₂ needed in FBR process
Flowrate of SiO₂ needed in Siemens process
Economic punishment when isolated photovoltaic system is used
Economic punishment when interconnected photovoltaic system is used
Quantity of interconnected solar panels
Quantity of isolated solar panels
Target amount of interconnected solar panels
Target amount of isolated solar panels
Economic incentive in isolated photovoltaic solar system (Response)
Economic incentive in interconnected photovoltaic solar system (Response)
Saleₙ-Si  Sale of silicon

Parameters

Sie  Unit fixed cost for silicon production in Siemens process
a₈₄O₂  Unit fixed cost for SiO₂ production
b₈₄O₂  Unit variable cost for silicon production in Siemens process
b₈₄SiCl₄  Unit variable cost for SiCl₄ production
c₈₄O₂  Exponent to consider the economies of scale for SiCl₄ production in Siemens process
c₈₄SiO₂  Exponent to consider the economies of scale for SiO₂ production
Eₜ₈₉  Demand of electricity in residential sector
Pₘ₉₆₉  Maximum production of silicon in Siemens process
Pₘ₉₆₆₄  Maximum production of SiO₂
E₉₉R  Base economic incentive in isolated solar panels
E₉₉R  Base economic incentive in interconnected solar panels
k₉₉  Factor used to annualize capital costs
pₙ₉₉  Proposed number of interconnected solar panels
pₙ₉₉  Proposed number of isolated solar panels
u  Parameter used in disjunction when production solar panels is equal to proposed
UOCₙ₈₃  Unit operation cost of silicon in Siemens process
UOCₙ₈₃  Unit operation cost for production of SiO₂
USₙ₈₃  Unit sale cost of silicon
UTCₙ₈₃  Unit transport cost of C to FBR process
UTCₙ₈₃  Unit transport cost of C to Hybrid process
UTCₙ₈₃  Unit transport cost of C to Siemens process
UTCₙ₈₃  Unit transport cost of silicon from FBR process to
consumption. Another process is the Fluidized Bed Reactor that saves between 80–90 percent of the energy consumed compared to the Siemens process [9] but the operating conditions are more difficult to achieve. It has been proposed different alternatives, one of them is an intensified FBR’s process by substituting the conventional reactors and separation zone with a reactive distillation column to improve the chemical conversion. An additional alternative process is based on both, the Siemens and the conventional FBR attempting to reduce the use of raw material [10]. The goals of smart manufacturing, sustainable production and circular economy could be realized through proper application of process intensification [11].

Solar energy is abundant, and its depletion is unlikely. The generation of solar photovoltaic systems does not cause any type of pollution and requires no energy consumption. However, the production of solar photovoltaic equipment produces wastewater, gas emissions, and solid wastes [12]. Because of that reason, it is necessary to propose a correct planning since obtaining the raw material to the final consumer to achieve better management of all involved components. Planning is a tool that has been used because it considers the most important aspects in the studied problems, resulting a better organization, and it is essential to take into account the economic point of view [13]. Through the supply chain optimization, it is possible to find better profits in the involved problems considering production, storage and transportation to maximize profit [14]. Also, planning is a form of exploration that can be adopted to show plausible futures [15]. Solving the scheduling problem in a shorter horizon and establishing a communication between planners and schedulers through inventory policies leads to novel integrated planning and scheduling models. With simultaneous optimization of inventory policies can reduce the implementation barriers in decision support systems and can improve the communication between planners and schedulers [16]. It is important to understand the impact of polysilicon industry in a city and the effect on its economy through simulation. The advantage of using simulation is to forecast the trends in future years according certain parameters [17]. An optimal photovoltaic energy storage system allows reducing voltage deviation, flicker, power loss, and linear load conditions in the distribution network, and it can provide a more intuitive income display for users in different regions and with different needs [18]. The photovoltaic supply chain involves multiple stakeholders, such as producers, collective schemes, consumers, and recyclers [19]. Studying the relationship between competing supply chains of photovoltaic industry and government with different policies results in better performance by those involved [20], being necessary to identify the role of the government in it [21]. Having budgetary constraints help to develop the supply chain [22]. Also, it was investigated the business dynamics of the critical decision-making factors regarding the multiple photovoltaic supply chains [23], one of the factors is how different materials help to increase the efficiency of solar panels [24].

The behavior of the population under various circumstances has not been addressed in a proper way. Considering the human behavior is especially important due to the high dependence of the energy system on it since the demand for each type of energy depends on the preference of the end user (population). It is possible to modify certain aspects that the consumer will respond according to the necessities that the government has addressed in a proper way. Considering the human behavior is essential to take into account the economic point of view [13].

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The behavior of the population under various circumstances has not been addressed in a proper way. Considering the human behavior is especially important due to the high dependence of the energy system on it since the demand for each type of energy depends on the preference of the end user (population). It is possible to modify certain aspects that the consumer will respond according to the necessities that the government considers pertinent. Analysts have been interested in the environmental determinants of why behaviors are allocated to particular choice...
alternatives for some time. Matching law is an analytical tool in the
descriptions of behavior-environment interactions [25]. Relatively dense
sources of reinforcement will feature relatively higher rates of behavior
(i.e., organisms demonstrate preference for the most reinforcing event-
ts/settings); this way, behavior matches reinforcement. To explain the
Matching law (Fig. 1), it was achieved through an experiment with pi-
geons in programs of intervals of concurrent variables. The pigeons were
presented with two buttons, each of which had different food reward
rate. Pigeons tended to bite or select the button that produced the
reward faster and more frequently than the other button. And the pro-
portion of its rate coincided with the proportion of its reward rates in the
two buttons. Animals may not seem appropriate to relate with human
behavior, but different studies have proved that this particular behavior
of reward and punishment can be linked with people as it shown by Reed
and Kaplan [26]. The implications of the matching law regarding the
matching law allows the producer to know the preferences of users under various
proposals and thus be able to predict what the reaction of the entire
system will be under the decision taken. In this way, the stakeholder is
proposed to implement a mathematical model that involves behavior through
matching law and planning of an important value-added compound.

Considering the behavior of the population through the matching
law allows the producer to know the preferences of users under various
proposals and thus be able to predict what the reaction of the entire
system will be under the decision taken. In this way, the stakeholder is
declared to determine which product will be the most attractive for con-
sumer; besides, the matching law allows government to know and
understand behavior of people in a timely manner to different problems
such as the use of alternative energies and this helps the final choice of
the consumer.

The main difference with the works mentioned before is that plan-
nings of photovoltaic systems do not consider users in the system, they
only include ways to evaluate costs or improve the processes but not to
take into account how the decision of a person could affect the whole
system. Also, it has not been studied the behavior of the population
through the matching law together with the strategic planning of solar
gate silicon (Si\textsubscript{SG}). This work incorporates optimization and human
behavior together, an interesting approach that has not been addressed
before. Silicon is used in the construction of solar panels to meet the
electrical energy demands of the residential sector and thus promoting
the use of solar energy of a given case study. The use of matching law is
related to the economic flow of incentives and penalties that will
influence the consumer's decision to select the type of solar panel to use
depending on the addressed case. Matching law provides a good tool to
model consumer response to incentives. This allows optimizing the
supply chain and planning of poly-silicon used to yield solar panels
involving producers, government and end-users.

2. Problem statement

The addressed problem seeks to meet the demands of electric power
of the residential sector. Due to the lack of use of solar energy, where its
generation capacity is not taken full advantage, it is necessary to pro-
mote the use of solar panels. One of the main limitations is the pro-
duction and distribution of the raw material to produce solar grade silicon [10], which can be implemented through the Siemens processes,
intensified FBR Union Carbide and a combination of both (Hybrid). This
solar grade silicone is the main component for producing solar panels.
The leading contribution of this paper is the implementation of the
matching law together with the planning of the solar grade silicon taking
into account maximum profits. Then, the problem addressed in this
paper can be stated as follows.

Given:

- Maximum available flowrates of raw materials
- Maximum capacity of production for each process:
  - Siemens
  - Intensified FBR
  - Hybrid
- Required composition for each process
- Conversion factors for:
  - Raw material per silicon produced, α
  - Silicon needed per solar panel, β
  - Electricity produced per solar panel, θ
- Electricity demand of residential sector
- Target panels
  - Established by the decision maker who is the person that pays or
    the person who has the ability of making the final decision
- Proposed panels
  - Established by the CEO of the company or the final user according
to the case
- Base economic incentives
  - Established by the decision maker
- Unit costs of operation and transport

![Fig. 1. Schematic representation for the idea of Matching Law.](image-url)
Matching law is adapted in a mathematical optimization model and the problem consists in determining the preference of the end user at a maximum profit. The way it is introduced is shown in Fig. 2. In the presented case study, the Government is the decision maker because it is responsible to pay incentives or implement economic punishments. The considered perspective is to study the effect of the government over producers and consumers. There are different studied scenarios, where it is intended to analyze those perspectives.

The consumer can choose between two options:

- Use solar panels interconnected to the network
- Use isolated solar panels

The final decision will be influenced by the incentives and punishments that the government will provide. Following those ideas, the behavior can be predicted.

Another important topic is the inclusion of different processes to obtain silicon. It was taken into account the Siemens process that is the most common process, but it has low yield. The fluidized bed reactor (FBR) was also used, and it has been intensified by using a reactive distillation column. This way, it has been achieved the development of new equipment in order to obtain better results [27]. Another type of process is a hybrid process combining both the Siemens and the conventional FBR. These processes are intended to reduce the use of raw material [10] in order to have intensified processes.

3. Optimization model

The proposed mathematical model is based on the superstructure shown in Fig. 3, which represents the problem to be addressed and involves all the possibilities to solve it. It considers the raw material available for using in different cities of the country. The raw material is transported to the different processes to produce solar grade silicon (Siemens, intensified FBR Union Carbide and Hybrid). This raw material must meet the composition requirements for its processing. Once the SiO₂ is obtained, it is used to produce solar panels isolated or solar panels interconnected to the existing network and will subsequently be used to meet the electricity demands of the country’s residential sector.

3.1. Raw material

The production capacity of the raw material \( F_{p}^{\text{max-SiO}} \), in this case silicon dioxide, cannot be greater than the maximum production capacity \( F_{p}^{\text{max-SiO}} \). To determine the existence of the production plants, the respective binary variable \( y_{ps}^{\text{SiO}} \) is multiplied by the maximum available capacity:

\[
F_{ps}^{\text{max-SiO}} \leq y_{ps}^{\text{SiO}} \cdot F_{ps}^{\text{max-SiO}}, \forall ps
\]

(1)

It is also necessary to know the greatest flow of each raw material \( F_{ps}^{\text{SiO}} \) that will serve as the required installation capacity:

\[
F_{ps}^{\text{max-SiO}} \geq y_{ps}^{\text{SiO}} \cdot F_{ps}^{\text{max-SiO}}, \forall ps, \forall t
\]

(2)

The cost of production of each of the raw material plant \( C_{ps}^{\text{SiO}} \) is calculated through the cost of operation \( UOC_{ps}^{\text{SiO}} \) plus the cost of capital \( \left( a^{\text{SiO}} + b_{ps}^{\text{SiO}} \cdot \left( F_{ps}^{\text{max-SiO}} \right)^{2} \right) \):

\[
C_{ps}^{\text{SiO}} = UOC_{ps}^{\text{SiO}} + b_{ps}^{\text{SiO}} \left( F_{ps}^{\text{max-SiO}} \right)^{2}, \forall ps
\]

(3)

In the same way for each raw material required (see Supportive information).

3.1.1. Distribution of raw material

The produced SiO₂ \( F_{ps}^{\text{SiO}} \) can be sent to the Siemens process \( F_{ps}^{\text{SiO}} \), to the FBR process \( F_{ps}^{\text{SiO}} \) or to the hybrid process \( F_{ps}^{\text{SiO}} \) likewise for each of the required raw material:

\[
F_{ps}^{\text{SiO}} = \sum_{x} F_{ps}^{\text{SiO} - \text{Si}}, + \sum_{f} F_{ps}^{\text{SiO} - \text{FBR}}, + \sum_{h} F_{ps}^{\text{SiO} - \text{Hyb}}, \forall ps, \forall t
\]

(4)

\[
F_{ps}^{c^{\text{SiO}}} = \sum_{x} F_{ps}^{c^{\text{SiO}} - \text{Si}}, + \sum_{f} F_{ps}^{c^{\text{SiO}} - \text{FBR}}, + \sum_{h} F_{ps}^{c^{\text{SiO}} - \text{Hyb}}, \forall pc, \forall t
\]

(5)

\[
F_{ps}^{\text{SiO}} = \sum_{x} F_{ps}^{\text{SiO} - \text{Si}}, + \sum_{f} F_{ps}^{\text{SiO} - \text{FBR}}, + \sum_{h} F_{ps}^{\text{SiO} - \text{Hyb}}, \forall phc, \forall t
\]

(6)

\[
F_{ps}^{\text{SiO}} = \sum_{x} F_{ps}^{\text{SiO} - \text{Si}}, + \sum_{f} F_{ps}^{\text{SiO} - \text{FBR}}, + \sum_{h} F_{ps}^{\text{SiO} - \text{Hyb}}, \forall ph, \forall t
\]

(7)

\[
F_{ps}^{\text{SiO}} = \sum_{x} F_{ps}^{\text{SiO} - \text{Si}}, + \sum_{f} F_{ps}^{\text{SiO} - \text{FBR}}, + \sum_{h} F_{ps}^{\text{SiO} - \text{Hyb}}, \forall psi, \forall t
\]

(8)

3.1.2. Transport cost of raw material

The transport cost \( C_{ps}^{\text{SiO-raw}} \) is calculated by multiplying the unit price of the transported raw material by the amount transported:

\[
C_{ps}^{\text{SiO-raw}} = \sum_{x} F_{ps}^{\text{SiO} - \text{Si}}, + \sum_{f} F_{ps}^{\text{SiO} - \text{FBR}}, + \sum_{h} F_{ps}^{\text{SiO} - \text{Hyb}}, \forall ps, \forall t
\]
cost of transport from each production plant of each raw material to the different processing plants (UTC_{S_{S_{Si}}}^q) to obtain S_{SG} times the flow of raw material to be transported (f_{S_{Si},p}^q): 

\[
\text{Cost}_{\text{transport}} = \sum_{q} \left[ \sum_{p} \left( \text{UTC}_{S_{S_{Si}}^q} \cdot f_{S_{Si},p}^q \right) + \sum_{f} \left( \text{UTC}_{C_{S_{Si}}^q} \cdot f_{C_{Si},f}^q \right) + \sum_{h} \left( \text{UTC}_{H_{S_{Si}}^q} \cdot f_{H_{Si},h}^q \right) \right]
\]

The transport cost of S_{SG} (Cost_{S_{SG}}) is calculated by multiplying the unit cost of transportation of the S_{SG} obtained in each of the different processes (UTC_{S_{Si}}^q) sent to the plants for the production of solar panels times the flow of silicon required (f_{S_{Si},p}^q): 

\[
\text{Cost}_{S_{SG}} = \sum_{q} \left[ \sum_{p} \left( \text{UTC}_{S_{S_{Si}}^q} \cdot f_{S_{Si},p}^q \right) + \sum_{f} \left( \text{UTC}_{C_{S_{Si}}^q} \cdot f_{C_{Si},f}^q \right) + \sum_{h} \left( \text{UTC}_{H_{S_{Si}}^q} \cdot f_{H_{Si},h}^q \right) \right]
\]

### 3.3. S_{SG} production process

Solar grade silicon (S_{SG}) is typically used in photovoltaic applications, and it is commercially manufactured by Siemens process since it is cheap and readily available. However, due to the low yield of the Siemens process (F_{S_{SG}}^{Si}), two alternative types of S_{SG} production processes have been considered. The first one is an intensified Fluidized Bed Reactor (FBR) process (F_{S_{SG}}^{FBR}) using a reactive distillation column. The second one is a hybrid process (F_{S_{SG}}^{Hyb}) combining the Siemens and conventional FBR processes to make the most of the advantages of both. To see a more detailed description of the considered processes, please consult the work by Ramírez-Márquez et al. [18]

The necessary raw material is sent to each of the processes:

\[
F_{S_{SG}}^{Si} = \sum_{p} f_{Si,p}^q + \sum_{f} f_{Si,f}^q + \sum_{h} f_{Si,h}^q, \forall s, \forall t
\]

Required composition of raw material should be considered to ensure that reactions of each process take place. The following relationships (12–15) indicate the relationship that must exist between raw material to produce S_{SG} in the case of Siemens process, where X represents the fraction of each compound.

\[
f_{S_{Si},p}^{Si} = X_{S_{Si}}^{Si} \cdot f_{S_{Si},p}^q, \forall p, s, \forall t
\]

\[
f_{C_{Si},f}^{Si} = X_{C_{Si}}^{Si} \cdot f_{C_{Si},f}^q, \forall f, s, \forall t
\]

\[
f_{H_{Si},h}^{Si} = X_{H_{Si}}^{Si} \cdot f_{H_{Si},h}^q, \forall h, s, \forall t
\]

Similar relationships applied for the FBR and Hybrid processes, as it can be seen in the supporting information section.

The S_{SG} obtained from the Siemens process (F_{S_{SG}}^{Si}) is calculated by multiplying the flow of raw material (F_{S_{SG}}^{Si}) times a conversion factor (X_{S_{SG}}^{Si}), similarly for the FBR and Hybrid processes:

\[
F_{S_{SG}}^{Si} = X_{S_{SG}}^{Si} \cdot F_{S_{SG}}^{Si}, \forall s, \forall t
\]

\[
F_{S_{SG}}^{FBR} = X_{S_{SG}}^{FBR} \cdot F_{S_{SG}}^{FBR}, \forall s, \forall t
\]

\[
F_{S_{SG}}^{Hyb} = X_{S_{SG}}^{Hyb} \cdot F_{S_{SG}}^{Hyb}, \forall s, \forall t
\]

The installation capacity (F_{S_{SG}}^{cap}) cannot be greater than the maximum production of S_{SG} (F_{S_{SG}}^{MAX}) and to determine its existence, the maximum available capacity is multiplied by the binary variable (Y_{S_{SG}}^{cap}):

\[
F_{S_{SG}}^{cap} \leq F_{S_{SG}}^{MAX} \cdot Y_{S_{SG}}^{cap}, \forall s, \forall t
\]

Furthermore, the flowrate of S_{SG} (F_{S_{SG}}^{MAX}) cannot be greater than the established capacity.

\[
F_{S_{SG}}^{cap} \geq F_{S_{SG}}^{MAX}, \forall s, \forall t
\]

The production costs of the S_{SG} (Cost_{S_{SG}}^{S_{SG}}) are calculated from the unit costs of operation (UOC_{S_{SG}}^{S_{SG}}) plus the capital costs as shown below (a_{S_{SG}}^{S_{SG}} + b_{S_{SG}}^{S_{SG}} F_{S_{SG}}^{cap}):

\[
Cost_{S_{SG}}^{S_{SG}} = \sum_{s} \sum_{t} UOC_{S_{SG}}^{S_{SG}} \cdot F_{S_{SG}}^{S_{SG}} + k_{t} \sum_{s} a_{S_{SG}}^{S_{SG}} \cdot Y_{S_{SG}}^{cap} + b_{S_{SG}}^{S_{SG}} \cdot F_{S_{SG}}^{cap} Y_{S_{SG}}^{cap}
\]

Similarly, for the FBR process and Hybrid processes (see Supportive information).

#### 3.3.1. Distribution of S_{SG}

The S_{SG} that is generated through the different processes (F_{S_{SG}}^{S_{SG}}),
Proposed superstructure.

The sale of SiSG (Sale^{Si}) is obtained by multiplying the unit sale price (US^{Si}) times the SiSG flow required of each process (P^{Si}_{fj}, F^{Si}_{fj}, P^{Si}_{ht,1}).

\[
\text{Sale}^{Si} = \sum_U \left( \text{US}^{Si} \left( \sum_{f,j} P^{Si}_{fj} + \sum_{h,t} F^{Si}_{ht,1} + \sum_h P^{Si}_{ht,1} \right) \right)
\]

Photovoltaic solar system

The flow of silicon to produce solar panels for the isolated photovoltaic system (R^{Si}_{pt,1}) was met from the Siemens (F^{Si}_{pt,1})))), FBR (f^{Si}_{pt,1})), and Hybrid processes (P^{Si}_{pt,1}):.

\[
E^{Si}_{pt,1} = \sum_{f,j} E^{Si}_{fj,1} + \sum_{h,t} E^{Si}_{ht,1} \forall p, t
\]

Similarly, in the case of the photovoltaic system interconnected with the network (P^{Si}_{pt,1}):.

\[
E^{Si}_{pt,1} = \sum_{f,j} E^{Si}_{fj,1} + \sum_{h,t} E^{Si}_{ht,1} \forall p, t
\]

The number of isolated solar panels according to the amount of required SiSG is determined by multiplying the silicon flow (R^{Si}_{pt,1}) by a factor (β) (for each panel per amount of silicon):

\[
P^{Si}_{pt,1} = \beta \cdot R^{Si}_{pt,1} \forall p, t
\]

The same in the case of solar panels interconnected with the network (P^{Si}_{pt,1}):.

\[
P^{Si}_{pt,1} = \beta \cdot R^{Si}_{pt,1} \forall p, t
\]

The electricity produced in the isolated photovoltaic system (E^{Si}_{pt,1}) is calculated by multiplying a conversion factor (β) by the silicon flow required for the construction of the solar panel (P^{Si}_{pt,1})..

\[
E^{Si}_{pt,1} = \beta \cdot P^{Si}_{pt,1} \forall p, t
\]
areas being commercially more difficult than others that is why the stakeholder sets the target considering the aforementioned. In addition, the producer is asked to propose a certain amount that he considers he can meet, knowing that by meeting that amount he will obtain an economic incentive, but if he does not meet it, he will be financially punished. In this way, both the stakeholder and the producer are involved. It is necessary to involve the receiver in establishing the targets to assure that they will try their very best in order to achieve the production target and earn the incentives but also they need to be aware of their limits and do not propose more than they can do, that is why punishments are also included.

\[ P_{pis}^{*} \] represents the number of panels proposed by the CEO of the company or the final user, according to the case. Those are the panels that CEO/user mean to achieve, it is a hypothetical situation because that number represents the number of panels that is easy to use for them but also the number must be reliable and achievable. They commit to reach the number and based on that the incentives/punishments are calculated. \( P_{pis}^{*} \) is the target quantity of solar panels established by the stakeholder. It is known that the economic situation throughout a considered territory could not be the same. The per capita income of each location is used to make difference in the demands according to their economic capabilities. IE\( \tilde{R} \) is the base economic incentive that can be obtained. \( u^k, \chi^k \) and \( v^k \) are required parameters.

- \( Y_{b,\text{out}}^{\text{IE}} \) will be activated if the number of proposed panels is equal to the number of proposed panels \( P_{pis}^{*} \). The reward is the base economic incentive and there is no punishment.
- \( Y_{b,\text{out}}^{\text{IE}} \) will be activated if the number of proposed panels is greater than the proposed panels. The reward will be greater than the base economic incentive and there is no punishment.
- \( Y_{b,\text{out}}^{\text{IE}} \) will be activated if the number of produced panels is lower than the proposed panels. There is no reward and the punishment will be greater than the base economic incentive.

\[
\begin{align*}
Y_{b,\text{out}}^{\text{IE}} & : P_{pis}^{*} = P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} > P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} > P_{pis}^{\text{ie}} - P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} < P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} < P_{pis}^{\text{ie}} - P_{pis}^{\text{ie}}
\end{align*}
\]

\[
\begin{align*}
P_{pis}^{\text{ie}} & = P_{pis}^{*} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} - \frac{P_{pis}^{*} - P_{pis}^{\text{ie}}}{2P_{pis}^{\text{ie}}} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} - \frac{P_{pis}^{*} - P_{pis}^{\text{ie}}}{2P_{pis}^{\text{ie}}}
\end{align*}
\]

\[
\begin{align*}
Y_{b,\text{out}}^{\text{IE}} & : P_{pis}^{*} = P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} > P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} > P_{pis}^{\text{ie}} - P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} < P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} < P_{pis}^{\text{ie}} - P_{pis}^{\text{ie}}
\end{align*}
\]

\[
\begin{align*}
P_{pis}^{\text{ie}} & = P_{pis}^{*} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} - \frac{P_{pis}^{*} - P_{pis}^{\text{ie}}}{2P_{pis}^{\text{ie}}} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} - \frac{P_{pis}^{*} - P_{pis}^{\text{ie}}}{2P_{pis}^{\text{ie}}}
\end{align*}
\]

The reformulated disjunction through a set of algebraic relationships can be seen in the supporting information section.

The matching law is used to relate the relative rate of response to the relative rate of reward in the use of isolated solar panels according to Borroto and Vollmer [28]:

\[
\frac{B_{pis}^{1}}{B_{pis}^{1} + B_{pis}^{2}} = \frac{R_{pis}^{1} - p_{pis}^{1}}{R_{pis}^{1} - p_{pis}^{1} + R_{pis}^{2} - p_{pis}^{2}} \forall pis, \forall \emptyset (34)
\]

3.7.2. Interconnected

In a similar way, the case of solar panels interconnected with the network is modeled as follows:

\[
\begin{align*}
Y_{b,\text{out}}^{\text{IE}} & : P_{pis}^{*} = P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} > P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} > P_{pis}^{\text{ie}} - P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} < P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} < P_{pis}^{\text{ie}} - P_{pis}^{\text{ie}}
\end{align*}
\]

\[
\begin{align*}
P_{pis}^{\text{ie}} & = P_{pis}^{*} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} - \frac{P_{pis}^{*} - P_{pis}^{\text{ie}}}{2P_{pis}^{\text{ie}}} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} - \frac{P_{pis}^{*} - P_{pis}^{\text{ie}}}{2P_{pis}^{\text{ie}}}
\end{align*}
\]

\[
\begin{align*}
Y_{b,\text{out}}^{\text{IE}} & : P_{pis}^{*} = P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} > P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} > P_{pis}^{\text{ie}} - P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} < P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} < P_{pis}^{\text{ie}} - P_{pis}^{\text{ie}}
\end{align*}
\]

\[
\begin{align*}
P_{pis}^{\text{ie}} & = P_{pis}^{*} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} - \frac{P_{pis}^{*} - P_{pis}^{\text{ie}}}{2P_{pis}^{\text{ie}}} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} - \frac{P_{pis}^{*} - P_{pis}^{\text{ie}}}{2P_{pis}^{\text{ie}}}
\end{align*}
\]

\[
\begin{align*}
Y_{b,\text{out}}^{\text{IE}} & : P_{pis}^{*} = P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} > P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} > P_{pis}^{\text{ie}} - P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} < P_{pis}^{\text{ie}} \\
Y_{b,\text{out}}^{\text{IE}} & : P_{pis} < P_{pis}^{\text{ie}} - P_{pis}^{\text{ie}}
\end{align*}
\]

\[
\begin{align*}
P_{pis}^{\text{ie}} & = P_{pis}^{*} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} - \frac{P_{pis}^{*} - P_{pis}^{\text{ie}}}{2P_{pis}^{\text{ie}}} \\
P_{pis}^{\text{ie}} & = P_{pis}^{*} - \frac{P_{pis}^{*} - P_{pis}^{\text{ie}}}{2P_{pis}^{\text{ie}}}
\end{align*}
\]
3.7.4. Interconnected case

\[
\begin{bmatrix}
Y_{\text{in},i}^F
\
R_{\text{pin},i}^F = \text{IE}^{R_i \cdot \text{in}} P_{\text{pin},i}^F \frac{C_{\text{pin},i}}{p_{\text{pin},i}}
\
R_{\text{pin},i}^2 = \text{IE}^{R_i \cdot \text{in}} \cdot \frac{C_{\text{pin},i}}{p_{\text{pin},i}}
\
p_{\text{pin},i}^2 = 0
\
R_{\text{pin},i}^2 = \text{IE}^{R_i \cdot \text{in}} \cdot \frac{C_{\text{pin},i}}{p_{\text{pin},i}}
\end{bmatrix}
\]

\[
Y_{\text{in},i}^F = \text{IE}^{R_i \cdot \text{in}} P_{\text{pin},i}^F \frac{C_{\text{pin},i}}{p_{\text{pin},i}} > p_{\text{pin},i}^F
\]

\[
R_{\text{pin},i}^2 = \text{IE}^{R_i \cdot \text{in}} \cdot \frac{C_{\text{pin},i}}{p_{\text{pin},i}}
\]

\[
R_{\text{pin},i}^2 = 0
\]

\[
R_{\text{pin},i}^2 = 0
\]

Likewise, the equations that involve behavior are modified as follows:

\[
B_{\text{pin},i}^1 + B_{\text{pin},i}^2 = \left( R_{\text{pin},i}^1 + R_{\text{pin},i}^2 - P_{\text{pin},i} \right) \forall p_{\text{pin},i}, \forall t
\]

\[
B_{\text{pin},i}^2 = \left( \text{IE}^{B_i \cdot \text{in}} P_{\text{pin},i}^F \frac{C_{\text{pin},i}}{p_{\text{pin},i}} \right) \forall p_{\text{pin},i}, \forall t
\]

\[
B_{\text{pin},i}^1 = \left( \text{IE}^{B_i \cdot \text{in}} \right) P_{\text{pin},i}^F \forall p_{\text{pin},i}, \forall t
\]

\[
B_{\text{pin},i}^1 = \left( \text{IE}^{B_i \cdot \text{in}} \right) P_{\text{pin},i}^F \forall p_{\text{pin},i}, \forall t
\]

3.8. Objective function

The considered objective function corresponds to the profit, which is calculated through the sale of $S_{\text{Br}}$ (Sale$^F$) minus all costs involved in the production and transport of the raw material and the different processes to obtain silicon:

\[
\text{Profit} = \left[ \begin{array}{l}
\text{Sale}^F - \text{Cost}^{\text{SiO}_2} - \text{Cost}^{\text{SiCl}_4} - \text{Cost}^{\text{HCl}} - \text{Cost}^{\text{SiCl}_4\text{-in}} - \\
\text{Cost}^{\text{SiCl}_4\text{-out}} - \text{Cost}^{\text{FBR}} - \text{Cost}^{\text{HCl}} - \\
\sum_{i} \left( R_{\text{pin},i}^2 - P_{\text{pin},i}^F \right) + \sum_{i} \left( R_{\text{pin},i}^1 - P_{\text{pin},i}^F \right)
\end{array} \right]
\]

4. Case study

The case study is applied in Mexico and the Government is the decision maker because it is responsible to pay incentives or implement economic punishments. This work is a first attempt to study planning and behavior together. It was decided to address the problem only from one stakeholder point of view. However, it will be interesting to consider multiple stakeholders for a future work to analyze the effect on the system. In this case, the perspective is considered from the government over the producers and consumers. There are different studied scenarios, where it is intended to analyze those perspectives. The purpose of exploring various scenarios is to understand the impact over the behavior of the final user together with the profit of the producer.

The raw material must be transported from its production point of the different plants according to the availability that INEGI [31] reports (Table 1). The location for the studied area is presented in Figure S1 in supporting information section. The data for the silicon production plans for either the Siemens, the intensified FBR or the Hybrid process are shown in Table 2. It was intended to promote work by considering local production and helping to increase consumption. For a future work, it will be recommended to incorporate a globalized supply chain to make the studied supply chain more attractive, as well as considering the variations for demands and costs of raw materials.

The required compositions are shown in Table 3. These data were obtained from the work by Ramírez-Márquez et al. [10] and it is necessary to meet those compositions. The processes consume energy mainly in the columns as shown in the supporting information section.

\begin{table}[h]
\centering
\caption{Cities that produce required raw material.}
\begin{tabular}{|c|}
\hline
\textbf{Raw Material} \\
\hline
C & Sabina, Coahuila  \\
& Tezoatlán de Segura, Oaxaca  \\
& San Martín, Sonora  \\
& San Pedro Corralitos, Chihuahua  \\
& Ojinaga, Chihuahua  \\
\hline
SiO$_2$ & Cananea, Sonora  \\
& Acapulco de Juárez, Guerrero  \\
& Agua Prieta, Sonora  \\
& Colima, Colima  \\
& Madera, Chihuahua  \\
& Santiago Papasquiaro, Durango  \\
\hline
H$_2$ & San Pedro Garza García, Monterrey  \\
& Ciudad de México  \\
\hline
SiCl$_4$ & Naucalpan de Juárez, México  \\
& Ejido Tepeyan, CDMX  \\
\hline
HCl & Cuautitlán Izcalli, México  \\
& Irapuato, Guanajuato  \\
& Ciudad de México  \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{Selected cities to install $S_{\text{Br}}$ production plants.}
\begin{tabular}{|c|c|c|}
\hline
\textbf{Siemens} & \textbf{FBR} & \textbf{Hybrid} \\
\hline
Ciudad de México & CDMX & CDMX  \\
Irapuato, Guanajuato & Guaymas, Sonora  \\
& Chihuahua, Chihuahua  \\
& Durango, Durango  \\
Madera, Chihuahua & Puebla, Puebla  \\
& Monterrey, Nuevo León  \\
\hline
\end{tabular}
\end{table}
The difference of energy consumption is related to the number of columns. Table 4 shows that the energy consumption for the hybrid process is between the Siemens and FBR processes, making it the best option due to different factors such as energy consumption and yield obtained [16].

For the installation of solar panels, 20 cities are selected [32], which represent the cities where entrepreneurship is being promoted in the country (Table 5). The contemplated time horizon is 30 years divided in periods of a year. The panels are installed during the first year.

It was considered one company for the production of solar panels that can be installed either in one city or in the 20 selected cities. Those are independent of the government, but they can accept incentives. The government gives economic incentives to promote solar panel production, but the profit is according to the company in all considered scenarios. Government only gives economic incentives. In addition, government takes care of the social welfare that is the reason to give economic incentives to promote solar production in order to have another option to produce electricity rather than burning fossil fuels.

Different scenarios were proposed, as it can be seen in Fig. 4.

- Scenario A: Government pays economic incentives and implements economic punishments to the CEO.
- Scenario B: CEO pays economic incentives to the final user.
- Scenario C: Government pays economic incentives and implements economic punishments to the CEO at the same time that CEO pays economic incentives to the final user.

According to the current consumption of electric energy [33], the necessary solar panels are determined if the total demand is covered, and this way the basis for subsequent calculations is obtained. Solar panels of 270 W are used.

Due to lack of information regarding to the number of dwellings inhabited by municipality, the data related to the number of inhabitants available on the INEGI platform are used [34]. The assumption made is that approximately 5 people live in each household.

The matching law is a mathematical approach that describes the relationship between the relative rate of response and the relative rate of reward in the face of a concurrent stimulus; from this law and respecting what is established, it is possible to give different interpretations and use it under diverse scenarios. The equation of matching law developed by Borrero and Vollmer [28] was adapted to the current problem statement. Producers (people who is involved in the production of solar panels) are rewarded or punished financially by the government according to the production of panels obtained in order to study their behavior. Under this background, it was determined the preference of population between isolated or interconnected solar panels.

In the case of using interconnected panels, the target panels established by the government represent satisfying 15 % of the national demand of the residential sector, it is important to highlight that the use of solar energy in Mexico to produce electricity only represents 0.3 % of installed capacity. The proposal by the producers corresponds to satisfy 10 % ($\pi_{\text{pin}}$) $\times$ 10. However, it should be considered that the economic situation is not the same throughout the country and cannot be demanded in a similar way. When the gross domestic product of each state is less than the average gross domestic product, the target panels ($\pi_{\text{pin}}$) that set the government correspond to 50 % less, that is, 7.5 % of the demand. On the other hand, if the gross domestic product of each state is greater than the average domestic product, the target panels that the government establishes would be 25 % higher, which corresponds to satisfying 18.75 % of the national demand for electricity. For the proposed panels, it is sought to satisfy 5 and 12.5 % when it is lower and higher, respectively.

Regarding to isolated solar panels, the target panels ($\pi_{\text{pin}}$) when the gross domestic product is lower correspond to satisfy 3.5 % of the demand, and when the gross domestic product is higher the target panels established by the government must satisfy 8.75 % of the demand. In addition, the panels proposed by the producers ($\pi_{\text{pin}}$) correspond to satisfying 5% of the demand for electricity for the residential sector. With respect to the proposed panels, 2.5 and 6.25 % in the smallest and largest case (see Table S4 in the supporting information section).

Final consumers need to propose consumption targets by type of solar panels because it is intended to involve the final consumer in order to study the reaction in the system. The way that the consumer proposes targets, allows imagine that one person is in charge of changes in certain city and this person is the voice of their city in order to establish solar panels target. The proposed values cannot be zero because the main idea is that the implementation of the supply chain involves solar panels, the main question is if these should be interconnected or not. The economic reward for the final consumer affects negatively to the profit of the company and the economic punishment of the consumer affects positively to the profit.

For scenario A, the base economic incentive is established from the ton of CO$_2$ produced per kWh in each type of panel and 5 dollars per ton of CO$_2$ are granted; based on this information, 4.86 dollars are granted for isolated panels (IE$^0$) and 2.916 dollars for interconnected panels (IE$^1$). In scenario B, the economic incentives are granted by the producers towards the population, that is, the end user, 4.86 $\times$ 10$^{-2}$ and 2.916 $\times$ 10$^{-2}$ $$/\text{panel for isolated and interconnected panels, respectively.}

5. Results

The optimization model is a mixed-integer nonlinear programming problem. The model consists of 1850 continuous variables and 1254 constraints. It was coded in the GAMS software and it was solved in a computer with an i7 processor at 3.2 GHz and 12 GB of RAM. The
average CPU time for each solution was around 0.25 s using the solver SBB. The solution has not been proven to be global optimal because the mathematical model is non-convex; however, different solvers were used and the one that generated better solutions was SBB.

The profit obtained in scenario A is $7.9188 \times 10^8$/y, where the profit only from the sale of Si$_{SG}$ corresponds to $2.41 \times 10^9$/y. The production costs of the raw material, as well as the different processes are shown in Table 6. The transportation cost of the raw material is $4.14 \times 10^8$/y and for silicon is $1.24 \times 10^7$/y.

Silicon production is mainly satisfied from the hybrid process (70.1%) as shown in Table S5 in the supporting information section. It corresponds with the results obtained by Ramírez-Márquez et al. [10] where conventional FBR had been intensified by changing the reactors and separation zone and replacing it with a distillation column. A hybrid process has been develop by combining both, Siemens and conventional FBR and incorporation the best of each one. Siemens process has the lowest cost but also the lowest silicon production rate. Intensified FBR has the highest cost but a large production. On the other hand, the hybrid process has the largest production rate with the middle production cost compared with the other two. It was shown that the Hybrid process is the most cost-effective one.

<table>
<thead>
<tr>
<th>Production cost, $/y</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>$5.57 \times 10^6$</td>
</tr>
<tr>
<td>H$_2$</td>
<td>$5.14 \times 10^7$</td>
</tr>
<tr>
<td>HCl</td>
<td>$3.66 \times 10^7$</td>
</tr>
<tr>
<td>SiCl$_4$</td>
<td>$9.35 \times 10^6$</td>
</tr>
<tr>
<td>SiO$_2$</td>
<td>$2.22 \times 10^6$</td>
</tr>
<tr>
<td>FBR</td>
<td>$4.34 \times 10^7$</td>
</tr>
<tr>
<td>Hybrid</td>
<td>$1.01 \times 10^8$</td>
</tr>
<tr>
<td>Siemens</td>
<td>$1.49 \times 10^7$</td>
</tr>
</tbody>
</table>

The profit obtained in scenario A is $7.9188 \times 10^8$/y, where the profit only from the sale of Si$_{SG}$ corresponds to $2.41 \times 10^9$/y. The production costs of the raw material, as well as the different processes are shown in Table 6. The transportation cost of the raw material is $4.14 \times 10^8$/y and for silicon is $1.24 \times 10^7$/y.

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The values B1 and B2 represent the flow of money that is directed to each option considered, it means that where there is greater flow of money is the largest number of households that select that type of panel. Eqs. 47 and 48 are established to calculate the number of households per municipality that choose the isolated or interconnected solar panels.

\[ X_{pis}^m = \frac{B_{pis}^1}{B_{pis}^1 + B_{pis}^2}, \forall pis, \forall pin, \forall t \]  

\[ X_{pin}^m = \frac{B_{pin}^1}{B_{pin}^1 + B_{pin}^2}, \forall pis, \forall pin, \forall t \]  

\( X_{pis}^m \) and \( X_{pin}^m \) times the number of houses in each city represent the number of houses that choose each type of panel.

In scenario A, the incentives affect positively and the penalties negatively the profit. Although there is a greater incentive for the use of isolated solar panels, results show that population is more inclined to use panels interconnected to the grid to produce electricity and to meet the demand for residential sector (See Figure S2 at supporting information section).

By exceeding the amount of proposed solar panels, the base incentive is obtained, and economic incentives are generated. Only Tlaxcala and Monterrey received economic punishments. The results are shown in the supporting information section (Table S6).

In addition, it can be noted that although the economy of the municipality is good or bad, as shown by the gross domestic product, users prefer to install interconnected solar panels over isolated solar panels (see Figure S3 in the supporting information section).

Scenario B only considers economic incentives granted by the producer towards final user. Eqs. 33 and 35 are similar than case A. In this case, there is no punishment because final user cannot be penalized. Incentives are given by the producer, so it is represented as an extra expense and negatively affects the profit. However, the results show a greater profit in scenario B (7.9223 × 10^8 $/y) than scenario A (7.9188 × 10^8 $/y), because scenario B does not consider economic punishment although the incentives granted are lower than the incentives given by government in scenario A.

Fig. 5 compares the behavior of the population in both scenarios, and it should be noted that the difference in selection in scenario B is smaller than in scenario A, and the tendency in both scenarios is to use interconnected panels. That means that by giving more incentives and punishment in scenario A exists more discrepancy in the final decision of the user; in contrast with scenario B where there is no punishment, the decision is closer between isolated and interconnected panels. It shows that modifying the economic incentives and punishments it is possible to control the behavior of the user. The intention of studying different scenarios with different decision-maker is to see the reaction of the supply chain.

Scenario C considers that the government grants economic incentives or economically punishes producers according to the panels proposed by the authorities and simultaneously the producers propose a target of solar panels for users to comply. In all three scenarios, profit is considered from the producer’s perspective. The incentives \( R_{pis}^1 \) given by government to the producers affect positively the profit but the incentives \( R_{pin}^2 \) given by the producer to the final user affect negatively the profit. When the profit is affected negatively by the incentives given by the producer to the final user means that final price of the solar panel will be less.

To model Scenario C, it is necessary to modify Eqs. 33 and 35 for isolated and interconnected cases as it is shown in Eqs. 39-44. Where the target of solar panels established by the government are equal to the target established by the producers. Also, the proposed panels by the producer are the same to the ones proposed by the user. The base economic incentive remains equal as in scenarios A and B, it means IE_{pis}^H is 4.86 and IE_{pin}^H is 4.86 × 10^-2 for isolated case, IE_{pis}^E is 2.916 and IE_{pin}^E is 2.916 × 10^-2 $/panel for interconnected case.

Population preference under scenario C is shown in Fig. 6, where the profit is 7.9214 × 10^8 $/y. In 40 % of the cities, more than 50 % of the users prefer to install isolated panels. In scenario A, 25 % of the cities prefer isolated panels. In contrast, in scenario B, all cities prefer to install interconnected panels.

It should be noted that in the addressed case study there were analyzed various scenarios to see how the whole system will react under different targets proposed by different entities and the presented
scenarios are only cases to show the applicability of the proposed approach. However, it can be easily modified in the mathematical model to study other scenarios with different targets because the model is general. This work is the first attempt to understand the relationship with matching law and planning under the fixed parameters. It can be noted that people prefer one option over the other because of the reward and punishment stablished. Producers do not really care about the type of panel, the difference will depend on preference of the final user, the geographical area, the required stability of the system, the budget of the user. It is important to say that those factors are not considered in the scope of the problem, but it will be worthy to include them in a future work.

The long-term projection is to recover the investment for installation in 30 years, because the capital cost is the most expensive. Also, flows of materials and rewards and all variables depend on time. The proposed model allows to consider different time periods. Finally, the proposed mathematical model is general and can be used under different scenarios with appropriate data.

6. Conclusions

This work has presented a mathematical optimization model capable of involving the conduct of people with the planning for the construction of solar panels isolated and interconnected with the network. It was considered since the distribution of the raw material for the production of solar grade silicon that can be obtained by Siemens, intensified FBR and Hybrid processes. Where intensified FBR and Hybrid processes are considered as intensified processes because they are improved to achieve higher production of silicon compared with the Siemens and conventional FBR processes. The proposed approach has been tested in a national case study in Mexico, taking into account the greater profit and preference towards the solar panels of the involved users.

Therefore, the hourly demand of electricity, hourly variation of solar irradiance, or the match of solar power with the hourly demand for electricity were not studied. Another problem that was not optimized is the selection of installation period. Those points need to be addressed in future works to obtain better, stronger, and more accurate solutions. Nevertheless, the solutions presented in this paper provide a good idea of the user’s behavior as a first attempt.

Matching law relates the relative rate of response and the relative rate of reward under a concurrent stimulus. Due to population is under continuous marketing, it is possible to promote usage of solar panels. Money has been used to motivate people and encourage them to do a better job through rewards and punishments. Even though the basis of matching law is with pigeons, it can be used in an optimization mathematical model. By using the matching law to study the human behavior, it is possible to predict preferences of population. Also, it can be created hypothetical scenarios to understand the main factors that change the final decision of the user and to help the stakeholder to decide according to their objectives and interests. In this case, it was proposed to involve economic rewards and punishments that will affect the final decision.

It should be noted in the presented case study that the profit remains similar between scenarios A, B and C (7.9188 × 10^8, 7.9223 × 10^8 and 7.9214 × 10^8 $/y). However, the difference lies in the behavior of the users, the preference for which they lean. Using models that predict behavior makes possible to control or manipulate the response expected from consumers under various established situations. Thanks to the analyzed study, it is possible to indicate that by changing the rewards, the behavior change.

The proposed model is general, and it can be applied to different case studies by making the appropriate changes in the involved parameters.

Declaration of Competing Interest

The authors report no declarations of interest.

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Appendix A. Supplementary data

Supplementary material related to this article can be found, in the online version, at doi:https://doi.org/10.1016/j.cep.2020.108241.

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